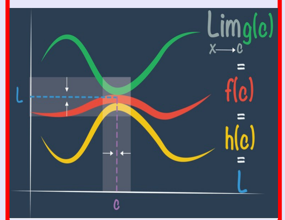
Math 261

Fall 2022

Lecture 28



For $y = f(x)$

when $f'(x) = 0$ or $f'(x)$ undefined

we get Critical Numbers.

If $f'(x) > 0$, we have increasing graph

If $f'(x) < 0$, we have decreasing graph

$f(x) = x^2 - 4x + 5$

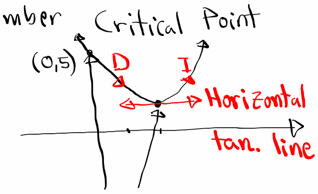
$f'(x) = 2x - 4$

$f'(x) = 0 \rightarrow 2x - 4 = 0$

$\boxed{x = 2}$ $(2, f(2)) = (2, 1)$

Critical Number Critical Point

x	$-\infty$	2	∞
$2(x-2)$	$-$		$+$
$f'(x)$	$-$		$+$
$f(x)$	$\searrow$		$\nearrow$



(2,1) is a minimum Point.

$$f(x)=\frac{x}{x^2+1}$$

$$f(-x)=\frac{-x}{(-x)^2+1}=\frac{-x}{x^2+1}=-\frac{x}{x^2+1}=-f(x)$$

Since $f(-x)=-f(x) \Rightarrow$ we have an odd function

odd functions are symmetric with respect to the origin.

$$f(0)=\frac{0}{0^2+1}=0 \qquad f'(x)=\frac{1(x^2+1)-x\cdot 2x}{(x^2+1)^2}$$

$$f'(x)=\frac{1-x^2}{(x^2+1)^2} \qquad f'(x)=0 \quad 1-x^2=0$$

$x=\pm 1$ C.N.

$f'(x)$ is defined everywhere

$(x^2+1)^2 \neq 0$

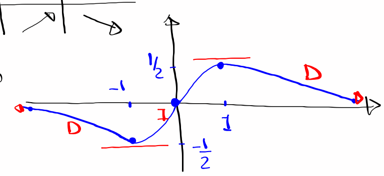
x	$-\infty$	-1	1	∞
$1-x$	$+$	$+$	$-$	
$1+x$	$-$	$+$	$+$	
$f'(x)$	$-$	$+$	$-$	
$f(x)$	$\searrow$	$\nearrow$	$\searrow$	

$f(1)=\frac{1}{2}$

$f(-1)=\frac{-1}{2}$

$$\lim_{x\to\infty} f(x)=\lim_{x\to\infty}\frac{x}{x^2+1}=0$$

$$\lim_{x\to-\infty} f(x)=0$$



$$f(x)=\frac{x-2}{x+1} \qquad \text{Domain} \Rightarrow x+1\neq 0 \quad x\neq -1$$

$(-\infty,-1)\cup(-1,\infty)$

Vertical Asymptote $x=-1$

x-Int $(2,0)$ $\rightarrow y=0 \rightarrow f(x)=0 \rightarrow \frac{x-2}{x+1}=0 \rightarrow x-2=0$ $\boxed{x=2}$

Y-Int $(0,-2)$ $\rightarrow x=0 \rightarrow f(0)=\frac{0-2}{0+1}=-2$

$f'(x)\neq 0$

$$f'(x)=\frac{1(x+1)-(x-2)\cdot 1}{(x+1)^2}=\frac{3}{(x+1)^2}$$

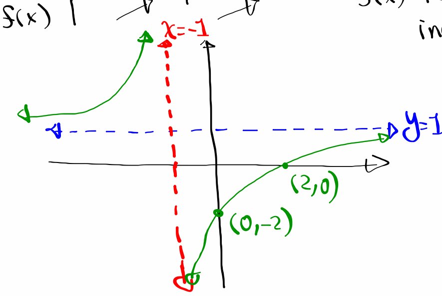
$f'(x)$ is undefined at $x=-1$.

x	$-\infty$	-1	∞
$f'(x)$	$+$	$+$	
$f(x)$	$\nearrow$	$\nearrow$	

$f'(x)>0$ C.N. $x=-1$

No C.P.

$f(x)$ is always increasing.

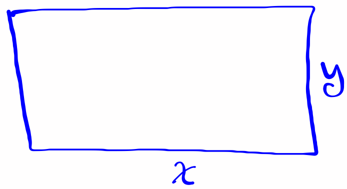


$$\lim_{x\to\infty} f(x)=\lim_{x\to\infty}\frac{x-2}{x+1}=1$$

$$\lim_{x\to-\infty} f(x)=1$$

H.A. $y=1$

Find dimensions of a rectangular garden with Perimeter 20 ft and maximum area.



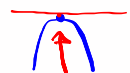
$2x + 2y = 20$

$x + y = 10 \rightarrow y = 10 - x$

$\text{Area} = xy$

$\text{Area} = x(10 - x)$

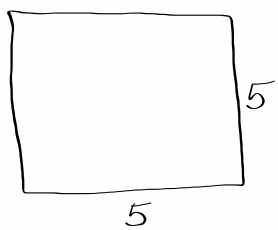
$A(x) = 10x - x^2$



$A'(x) = 10 - 2x$

$A'(x) = 0 \rightarrow 10 - 2x = 0$

$x = 5$



5 ft by 5 ft.

I have 300 ft of Fencing, and wish to make a closed rectangular garden next to a wall.

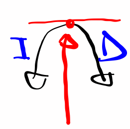
Find dimensions that gives us max. area.

$2x + y = 300$

$\text{Area} = xy$

$A(x) = x(300 - 2x)$

$A(x) = 300x - 2x^2$



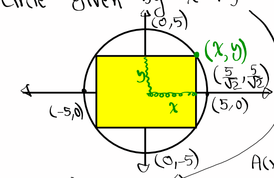
$A'(x) = 300 - 4x$

$A'(x) = 0 \qquad 300 - 4x = 0 \qquad \boxed{x = 75}$

Dimensions $\Rightarrow$ 75 ft by 150 ft.

(75,150)

Find dimensions of a rectangle inside of a Circle given by $x^2+y^2=25$ with max. Area.



Area $= 2x \cdot 2y$

Area $= 4xy$

$A(x) = 4x \cdot \sqrt{25-x^2}$

$A(x) = 4\sqrt{x^2(25-x^2)}$

$= 4(25x^2-x^4)^{1/2}$

$y^2 = 25-x^2$

$y = +\sqrt{25-x^2}$

$A'(x) = 4 \cdot \frac{1}{2}(25x^2-x^4)^{-1/2} \cdot (50x-4x^3)$

$= \frac{2\cdot(50x-4x^3)}{\sqrt{25x^2-x^4}} = \frac{4x(25-2x^2)}{\sqrt{x^2(25-x^2)}}$

$= \frac{4x(25-2x^2)}{x\sqrt{25-x^2}} = \frac{4(25-2x^2)}{\sqrt{25-x^2}}$

$A'(x)=0$ @ $25-2x^2=0$

$x = \frac{5}{\sqrt{2}}$ in QI

$A'(x)$ is undefined @ $25-x^2=0$ $x=5$ Not practical

$\frac{5}{\sqrt{2}} \approx 3.5$

x	$-\infty$	≈ 3.5	∞
$25-2x^2$	+	−	
$A'(x)$	+	−	
$A(x)$	↗	↘	

Max. Point

Max. Area @ $\left(\frac{5}{\sqrt{2}}, \frac{5}{\sqrt{2}}\right)$

Dimensions $\frac{10}{\sqrt{2}}$ by $\frac{10}{\sqrt{2}}$

Area $= \frac{10}{\sqrt{2}} \cdot \frac{10}{\sqrt{2}} = 50$

Use Quadratic approximation to find $\sqrt{5}$

$$Q(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2}(x-a)^2$$

From Calc.

$\sqrt{5} \approx 2.236$

$f(x) = \sqrt{x}$ $\quad f(4) = 2$

$a = 4$

$f'(x) = \frac{1}{2}x^{-1/2}$

$f''(x) = \frac{-1}{4}x^{-3/2}$

$f'(x) = \frac{1}{2\sqrt{x}}$ $\quad f'(4) = \frac{1}{2\sqrt{4}} = \frac{1}{4}$

$f''(x) = \frac{-1}{4x\sqrt{x}}$ $\quad f''(4) = \frac{-1}{4(4)\sqrt{4}} = \frac{-1}{32}$

$$Q(x) \approx 2 + \frac{1}{4}(x-4) + \frac{\frac{-1}{32}}{2}(x-4)^2$$

$$\sqrt{x} \approx 2 + \frac{1}{4}(x-4) - \frac{1}{64}(x-4)^2$$

near 4

$$\sqrt{5} \approx 2 + \frac{1}{4}(\cancel{5-4})^{1} - \frac{1}{64}(\cancel{5-4})^{2}\,{}^{1}$$

$$\approx 2 + \frac{1}{4} - \frac{1}{64} = \boxed{2.234}$$